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恐龙蛋壳的生物力学性质（III）——恐龙幼雏出壳时蛋壳破裂的力学分析

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摘要 恐龙幼雏出壳时并不象鸟类那样用喙向蛋壳施加集中载荷，而是施加分布载荷。根据已建立的微分方程，恐龙蛋壳的环向应力 σ_θ 始终大于沿母线方向的应力 σ_Φ 而成为壳内的最大应力。蛋壳破裂的力学判据是：$\sigma_\theta \geqslant \sigma_{破坏}$。本文研究了四种恐龙蛋在幼雏出壳时幼雏的挤压力所造成的蛋壳应力。结果表明四种恐龙幼雏出壳时首先出现沿母线方向的裂纹以造成蛋壳的断裂，其最大环向应力和沿母线方向的应力均出现在蛋壳的最大直径附近。以 *Ovaloolithus* 为代表的A型蛋的应力最小，最不容易破碎，幼雏出壳时，可能比较困难。

关键词 恐龙蛋，旋转薄壳，应力，蛋壳强度，出壳

一、前　言

恐龙以产卵的方式繁殖后代。卵的孵化受诸多因素的影响，其中比较重要的是蛋壳的强度。如果蛋壳的强度很低，则蛋在孵化期间内由于周围可能出现的某些外力作用而使蛋壳破裂。本世纪50、60年代，大量使用的DDT以及其它一些含氯农药直接影响了一些鸟类蛋壳的形成，使得这些蛋壳变薄，易于破碎，从而导致了这些鸟类濒于绝灭。另一方面，如果蛋壳的强度过高，幼雏不能顺利出壳，同样也不利于生物种的延续。本文主要根据生物力学的基本原理和方法对幼雏出壳时幼雏的挤压力所造成的蛋壳应力进行研究，以帮助我们理解蛋壳结构的演化及恐龙绝灭的原因。

文中研究的恐龙蛋化石参见赵资奎等（1994，表1）和马和中等（1994，表1）的文章。

二、恐龙蛋壳的平衡方程和受力分析

现有的资料表明，现生爬行动物的蛋壳较薄，大都是一层纤维质软膜。幼雏用卵齿勾

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破卵膜后很容易挤破蛋壳。鸟蛋壳相对较坚硬,雏鸟主要用喙先在蛋壳近钝端(一般在最宽处附近)啄一小孔,然后靠小鸟的蠕动将蛋壳挤破。恐龙的情形与现生爬行动物和鸟类有所不同。一方面,恐龙没有喙,幼雏是否发育有卵齿目前还不清楚;另一方面恐龙的蛋壳较厚。本文研究的四种恐龙蛋的壳厚度在1.3毫米至2.5毫米之间。因此恐龙幼雏出壳时并不象鸟那样用喙向蛋壳施加集中载荷,而是施加分布载荷。从力学角度来看,可采用薄壳无矩理论进行分析。

根据旋转薄壳无矩理论,可以建立微分方程组以求解沿母线方向的应力 σ_Φ 和环向应力 σ_θ (S. Timoshenko et al., 1959)。为求解方便,可先作一符合实际情况的假设:恐龙幼雏出壳时对蛋壳的作用力主要是沿壳面法线方向的垂直压力 $Z = p$,沿壳内面方向的作用力 $X = Y = 0$;同时压力分布对于几何对称轴是对称的,即: $N_{XY} = N_{YX} = 0$。由此,沿对称轴方向的平衡方程经简化并积分变为:

$$\sigma_\Phi = \frac{1}{hR_0}\int_0^\Phi \sigma_\theta h R_1 \cos\phi \, d\Phi \qquad (1)$$

式中 R_0 为蛋壳上研究点处的平行圆半径,R_1 为该点处母线的曲率半径,Φ 为该点法线与旋转轴之间的夹角(图1)。

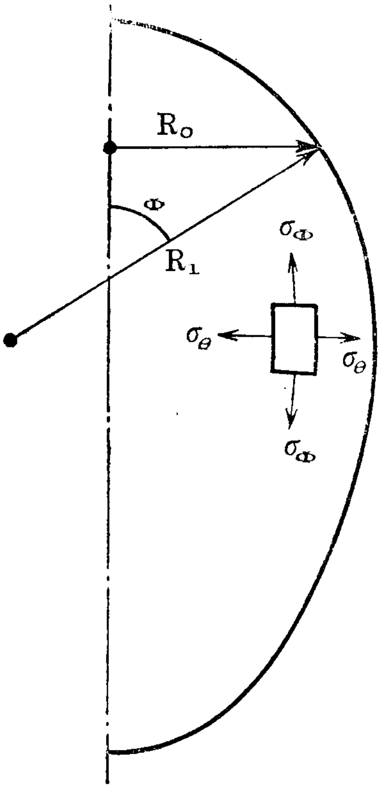


图1　蛋壳的几何参数与力学参数

Fig. 1　Geometric and mechanic data of dinosaur eggshells

蛋壳的法向平衡方程为:

$$\overline{R_{01}}\sigma_\Phi + \sigma_\theta = \overline{R_{0h}}p,$$

式中 $\overline{R_{01}} = R_0/R_1 \sin\Phi$,$\overline{R_{0h}} = R_0/h \sin\Phi$。可以看出,蛋壳厚度 h 越大或 R_0 越小,即 $\overline{R_{0h}}$ 越小,则壳内应力 σ_θ 和 σ_Φ 越小。上式经改写成为:

$$\sigma_\theta = \overline{R_{0h}}p - \overline{R_{01}}\sigma_\Phi, \qquad (2)$$

由(2)式可知,当 $\overline{R_1}$ 减小时,$\overline{R_{01}}$ 增大,方程中 $\overline{R_{01}\sigma_\Phi}$ 增大,从而使 σ_θ 减小。

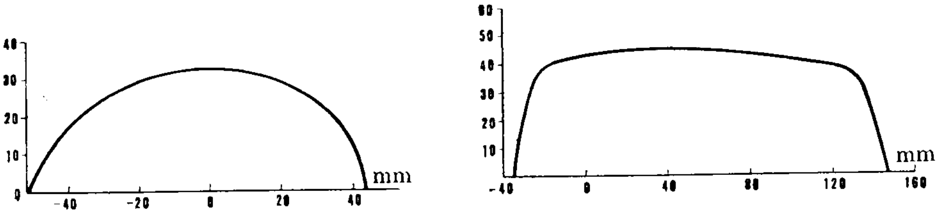
作为旋转薄壳结构,恐龙蛋壳的环向应力 σ_θ 始终大于沿母线方向的应力 σ_Φ 而成为壳内的最大应力。根据强度理论,使幼雏破壳孵出的力学判据是:

$$\sigma_\theta \geqslant \sigma_{破坏},$$

此处 $\sigma_{破坏}$ 是导致蛋壳破坏的拉伸应力。如果蛋壳厚度 h 越小,平行圆半径(即旋转半径)越大,母线越直(即 $\overline{R_1}$ 越大),则蛋壳越容易破裂,幼雏出壳越容易。反之,幼雏出壳比较困难。

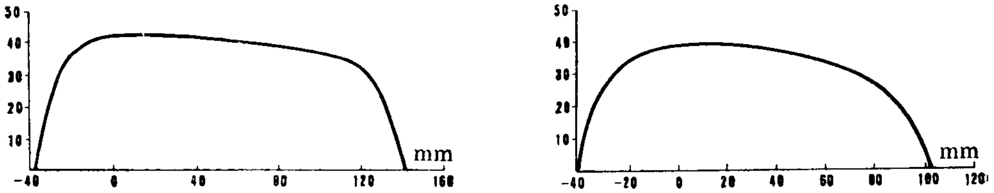
三、恐龙幼雏出壳时蛋壳中的应力数值解

将(1)、(2)两个方程联立，可以求得恐龙幼雏出壳对蛋壳施加压力p时壳中的应力



(1) A型蛋

(2) B型蛋

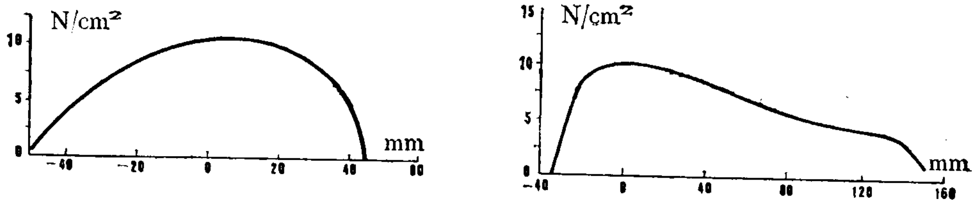


(3) C型蛋

(4) D型蛋

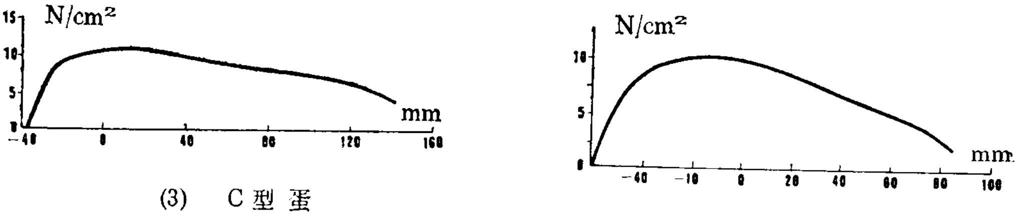
图2 恐龙蛋的外形图(仅绘出一半,另一半以长轴对称)

Fig. 2 The shape of dinosaur eggs (only up part)



(1) A型蛋

(2) B型蛋



(3) C型蛋

(4) D型蛋

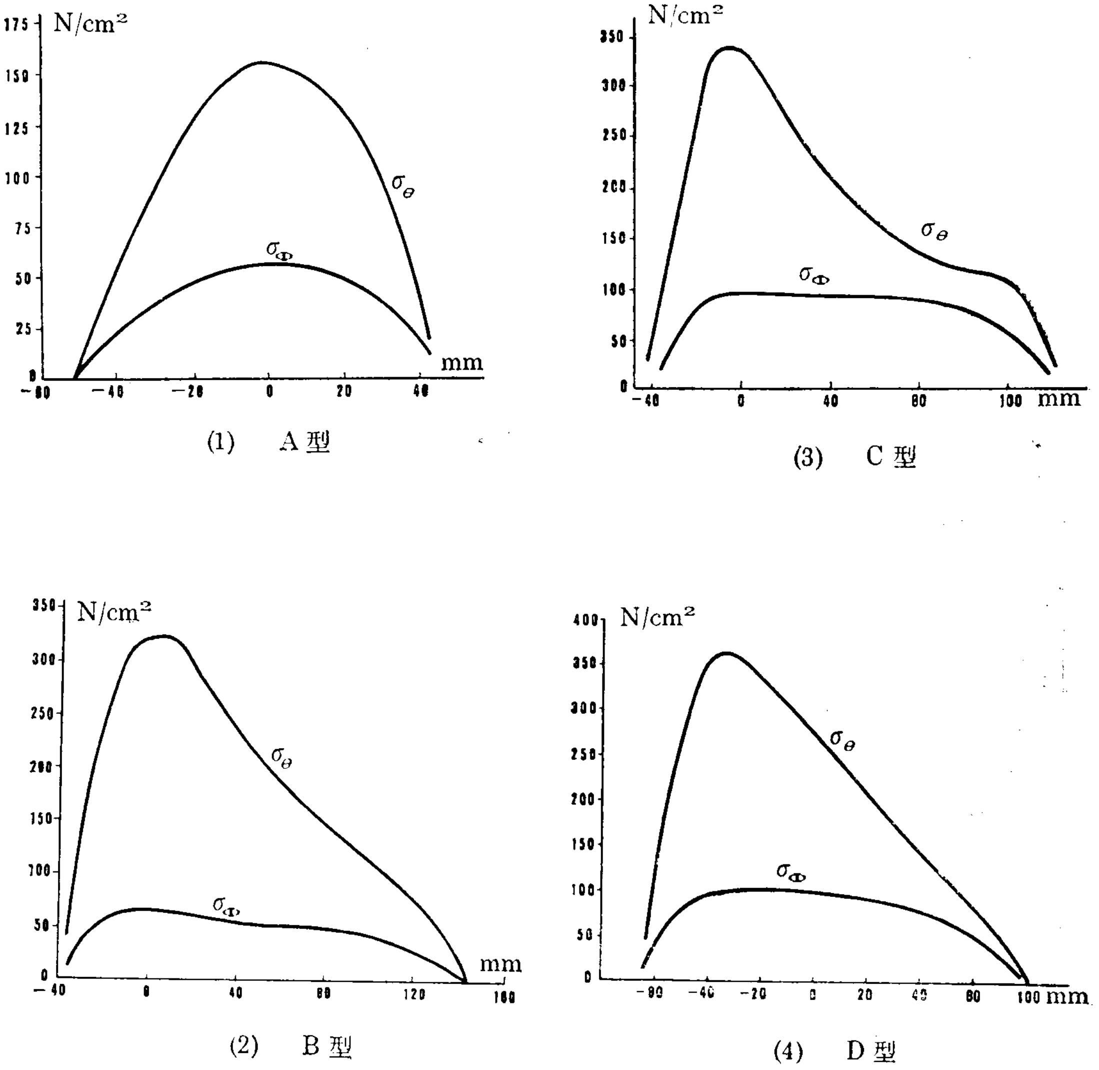
图3 恐龙蛋壳受到的出壳内压分布

Fig. 3 Distribution of inner pressure loaded by young dinosaurs

表 1　A、B、C、D 四种恐龙蛋蛋型的几何数据(图中单位均为毫米)

Tab. 1　Geometric data of four kinds of dinosaur eggs (mm)

蛋的型号 egg type	A	B	C	D
最大长半径 half of the maximal long axis	47	100	90	73
最大短半径 half of the maximal short axis	33	44	40	38
壳　　厚 eggshell thickness	2.4—2.6	1.4—1.8	1.4—1.7	1.2—1.4



(1)　A 型

(3)　C 型

(2)　B 型

(4)　D 型

图 4　恐龙蛋壳中的应力分布

Fig. 4　Distribution of stress of dinosaur eggs hells

σ_θ 和 σ_Φ 的值。由于蛋壳形状不规则，不能用一个解析函数表示，所以很难对(1)式直接积分而只能用数值方法借助于电子计算机来求解(李庆扬等，1986)。

表1列举了四种恐龙蛋的有关数据。四种恐龙蛋的外形如图2所示（图中只绘出了上半部分）。

为了反映恐龙幼雏出壳时的加力特征，可以在蛋壳内部施加了一组分布载荷。此载荷在中间较大，两端为零，沿长轴方向的合力为零，载荷沿长轴方向的分布如图3所示。

根据上述的四种恐龙蛋的几何数据和外力数据，经过计算求得应力 σ_θ 和 σ_Φ 的值，具体结果如图4所示。

四、结果讨论

根据计算分析，本文的结论如下：

1. 假设幼雏出壳时它们施加给蛋壳的压力相同，即各种蛋壳所承受的最大值均为10牛顿/厘米^2，那么A型恐龙蛋蛋壳的应力较小，最大应力仅为155牛顿/厘米^2，然而B、C、D三种恐龙蛋的蛋壳的最大应力约为350牛顿/厘米^2。从而表明应力 σ_θ 和 σ_Φ 随压力 p 的增加而增加，也随半径厚度比（R/h）的增加而增加。以 *Ovaloolithus chinkangkouensis* 为代表的A型恐龙蛋一般有两种保存状况：完整或完全破碎。杨钟健(1954)认为从保存状态来看，破碎的A型蛋壳基本上属于原地埋藏，似乎表示原来是一窝蛋。由于A型恐龙蛋的应力最小，最不容易破碎(赵资奎等，1994)，因此我们推测破碎的原因很可能是恐龙幼雏出壳造成的。

2. 四种恐龙幼雏出壳时最大应力 σ_θ 和 σ_Φ 均出现在蛋壳的最大直径附近。

3. 幼雏出壳时的环向应力 σ_θ 比沿母线方向的应力 σ_Φ 大得多，因此恐龙幼雏出壳时首先出现沿母线方向的裂纹以造成蛋壳的断裂。

在山东莱阳发现的一窝长形长形蛋 *Elongatoolithus elongatus*（赵资奎，1975），基本依原来的位置保存，大多数的蛋在钝端破裂(参看杨钟健1954，图4，图版I，1)。值得注意的是至少有8个蛋其破裂的部分仍留在这些蛋的附近。这不象是受了侵蚀而损坏的，唯一可能的解释就是代表孵化过的蛋(杨钟健，1954)。长形长形蛋的基本特征与B、C、D型恐龙蛋相似，本文的分析结果与杨钟健的意见一致。

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BIOMECHANICAL PROPERTIES OF DINOSAUR EGGSHELLS (III) ——THE MECHANIC ANALYSIS OF THE BABY DINOSAURS' EMERGING FROM THEIR EGGS

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Key words Dinosaur eggshell; Rotational thin shell; Stress; Eggshell strength; Hatching

Summary

The eggshell strength has a great influence upon hatchability. Generally, eggshells must be strong enough to withstand external pressure during incubation. It has been shown that the eggshell thinning in some kinds of birds as the result of intake of DDT or other chlorinated hydrocarbons is the cause why they were near extinction. On the other hand, eggshells need to be fragile enough to crack easily when the young hatch. If the eggshell strength is greater, it is hard for the young to emerge from its eggshell.

The purpose of this paper is to analyze the stress of dinosaur eggshell loaded by the hatchlings when they hatch, and four types of dinosaur eggshells are available (see Table 1 of Zhao et al., 1994; Table 1 of Ma and Zhao, 1994).

Most of the living reptiles have flexible, parchment-like eggshells composed of fibres and relatively small amounts of calcium carbonate. The hatchlings, each equipped with a small, sharp "egg tooth", crack their eggshells to emerge. However, the young birds poke at the blunt end of the egg with their beak, then struggle out from their eggs. The hatching process of baby dinosaurs seem to differ from that of the living reptiles and birds. The dinosaur eggshells are thicker, and it is uncertain whether the young dinosaur is equipped with an "egg tooth". Therefore, it is reasonable to think that they could exert distributive load instead of concentric load on the eggshells when they hatch. Zhao et al. (1994) advanced that the dinosaur eggshells can be considered as rotational thin shell. In this case, the circular stress σ_θ and the stress σ_φ producing along the generatrix direction on the eggshell can be calculated by the membrane theory (Timoshenko et al., 1959). For the sake of convenience, let us take a practical assumption: that the forces exerted by the young within the eggshell are along the normal direction, $Z = p$, the acting forces in the inner face of the eggshell, $X = Y = 0$; the pressure are symmetrically

distributed along the eggshell for the geometric symmetry axis. Hence the internal forces, $N_{XY} = N_{YX} = 0$. The equilibrium equation in the symmetric direction can be simplified and integrated into the following form:

$$\sigma_{\Phi} = \frac{1}{hR_0}\int_0^{\Phi} \sigma_{\theta} h R_1 \cos\phi \, d\Phi \, , \tag{1}$$

where, R_0 is the parallel circle radius at this point on the eggshell, R_1 is the radius of generating curve at this point, Φ is the angle between R_1 and the rotational symmetry axis, as shown in Fig. 1.

The equilibrium equation in the normal direction on the eggshell can be written as:

$$\overline{R_{01}}\sigma_{\Phi} + \sigma_{\theta} = \overline{R_{0h}}p,$$

in which $\overline{R_{01}} = R_0/R_1 \sin\Phi$, $\overline{R_{0h}} = R_0/h \sin\Phi$. It can be seen that σ_θ and σ_Φ are in inversely proportion to the eggshell thickness h, but in proportion to R_0 or R_{0h}. The above equation can be rewritten as:

$$\sigma_{\theta} = \overline{R_{0h}}p - \overline{R_{01}}\sigma_{\Phi}. \tag{2}$$

It shows that the minor R_1 leads to the rise of $\overline{R_{01}}$ and $\overline{R_{01}\sigma_{\Phi}}$, thus to the decrease of σ_θ.

The eggshell, as the rotational thin shell, is formed by a curve moving around a rotational axis. Its circular stress σ_θ is always greater than the stress σ_Φ producing along the generatrix direction, and becomes the maximal stress. According to the strength theory, the mechanic criterion of the eggshell broken by the young is:

$$\sigma_{\theta} \geqslant \sigma_{\mathrm{broken}}$$

where σ_{broken} is the drawing stress causing eggshell breakage. Assuming that the eggshell is thinner, the rotational radius, greater, and the generatrix, straighter, the eggshell is easier to crack. That is to say, the baby dinosaurs can easily crack their eggshell to emerge. On the contrary, it is hard for them to hatch.

From (1) and (2), we can calculate the values of σ_θ, σ_Φ on the eggshell under action of forces exerted by the young. Because of the irregular shape of the eggshell, numerical method is used to substitute the integration (Li et al., 1986).

In order to depict the pressure aspects on the eggshell loaded by the young, we put a group of distributive pressure against the inner face of the eggshell. It turned out that the pressures are greater in the middle portion of the egg, zero in both ends; the resultant of forces along the long axis of the egg is zero, too. The distribution of inner pressure loaded by the baby dinosaurs is shown in Fig. 3.

According to the given data, the stresses σ_θ, σ_Φ can be calculated and shown in Fig. 4.

Based upon the foregoing calculation, the following conclusions can be drawn:

1. Assuming that the pressure against the eggshells loaded by the baby dinosaurs is the same, i.e. the eggshells are all subjected to maximal pressure 10 N/cm^2, the maximal stress σ_θ is only 155 N/cm^2 in the type A of dinosaur eggshell, and about 350 N/cm^2 in the types B, C and D. It can be seen from Fig. 4 that the stresses σ_θ and σ_Φ increases with the pressure p, and with R/h, too. The type A, represented by *Ovaloolithus chinkangkouensis*, is completely preserved eggs or egg-fragments in situ without being transported (Young, 1954). Because the type A has

the low stress level, it is hard to crack under action of external pressure (Zhao et al., 1994). Therefore, we believe that the egg-fragments ***in situ*** were caused by hatchlings when they hatched.

2. The maximal stresses σ_θ, σ_Φ in the four types of dinosaur eggshell analyzed here are all arising near the point with the greatest diameter.

3. When the young hatches, the circular stresses σ_θ is much greater than the stress σ_Φ producing along the generatrix direction. Consequently, the crack in the eggshell, caused by the young, takes place along the generatrix direction.

A nest of the dinosaur eggs, identified as ***Elongatoolithus elongatus*** (Zhao, 1975), has been collected in the Upper Wangshih Series of Laiyang, Shandong Province (see Fig. 4 and plate I, 1 of Young, 1954). It is very interesting that at least eight of the eggs and many broken egg-fragments still lay in situ and concentrated at one end of the egg. It is unlikely that they were broken by erosion. The only cause why they were broken is that they have been hatched (Young, 1954). The geometric data of these eggs is similar to that of the types B, C and D. It is evident that the present result is consistent with Young's explanation.