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恐龙蛋壳的生物力学性质（I）
——在外力作用下恐龙蛋壳的应力分析

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关键词　恐龙蛋　薄壳　外压　应力

内　容　提　要

本文研究恐龙蛋在外压力作用下蛋壳内部的内力和应力变化规律，计算出蛋壳内部不同方位和不同方向上内力和应力的变化值。对四种不同类型恐龙蛋的蛋壳应力值进行了比较，得出了影响应力大小的因素和蛋壳在外压力作用下的破坏规律。

一、前　言

恐龙以产卵的方式繁殖，卵的发育是在陆地上的环境条件中进行的。从恐龙蛋化石埋藏的情形以及蛋化石在蛋窝中排列的方式来看，可以认为它们和其它的大多数爬行动物的蛋一样，主要是靠日光或环境的温度使卵发育，孵化期一般应比较长。

恐龙蛋最外层是一层坚硬的钙质壳，它主要的功能是保护卵不受外力的损伤，限制卵内水份的蒸发，保证胚胎发育时呼吸气体的交换以及在孵化后期幼雏能够破壳而出。因此，从力学的角度看，蛋壳结构的力学性质对恐龙蛋的孵化有着很大的影响。

近来的研究表明，根据恐龙蛋壳的几何结构形态，可以估算出恐龙胚胎在正常的发育情况下蛋壳对水蒸汽和呼吸气体的传导率，探讨恐龙蛋孵化期间蛋窝的微环境（Seymour 1979; Williams 等，1984; 牟耘，1992）。更有意思的是从恐龙蛋壳的其它研究发现，恐龙在中生代最后的绝灭可能与其繁殖有着密切的关系（Erben, 1970; Erben 等，1979；赵资奎，1978，1990，1994；赵资奎等，1991；Zhao 等，1993）。

不同类型的恐龙蛋，其大小、形状以及蛋壳的厚度和组织结构都有很大的变异（杨钟健，1954，1965；Zhao, 1993, 1994）。可以认为，这种形态学上的结构相对复杂性是由其功能适应所决定的。因此，从生物力学的角度来分析这些恐龙蛋壳的基本力学性能，对于探索恐龙蛋壳的演化，恐龙繁殖的生物学性质以及恐龙最后的绝灭可能具有重要的科学意义。

本文将研究四种类型的恐龙蛋壳在外界压力下内部的应力分布，确定这些恐龙蛋壳抵抗外界压力的能力。

二、材料和方法

晚白垩世后期的各类恐龙蛋壳的结构和功能基本上已接近鸟蛋壳的组织水平，是比较进步和比较完善的(Zhao,1993, 1994)。根据现有的条件和蛋壳的结构特性，考虑到本研究需要保存完整的蛋化石，笔者选取了在山东莱阳和广东南雄发现的属于这一地质时代的四种恐龙蛋化石：*Ovaloolithus chinkangkouensis*, *Macroolithus yaotunensis*, *Macroolithus rugustus* 和 *Nanhsiungoolithus chuetienensis* 等(杨钟健，1954，1965；赵资奎，1975，1979）作为研究材料。对每一种类型的蛋，笔者根据实际情况各选取最为完整的蛋化石一枚，并对其原有测量数值作了校正，见表 1。

从几何学观点来看，这些蛋壳均可看成是曲线 ABC（称为母线）绕 AB 轴线旋转而形成的旋转体，如图 1 所示。从力学的角

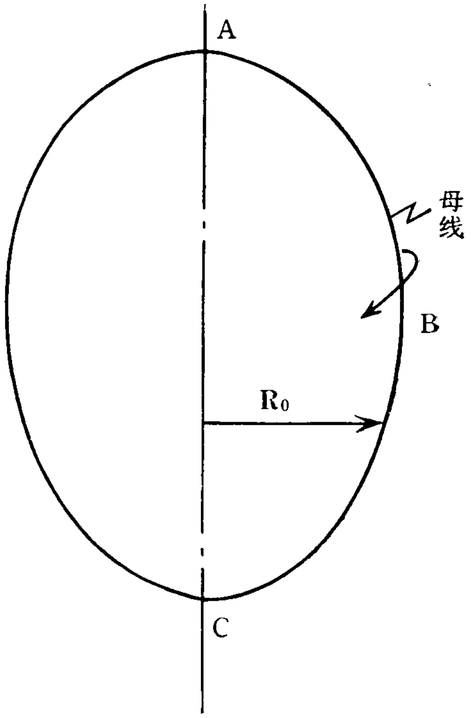


图 1　作为旋转薄壳的蛋壳结构

Fig. 1　The dinosaurian eggshell structure as thin shell of revolution

表 1　本文研究的蛋化石材料

Table 1　The dinosaur eggs analyzed in this study

蛋化石名称	简要特性	产地及层位	附记
Ovaloolithus chinkangkouensis	椭球形，完整蛋化石一枚(V732b) 长径=94 毫米，最大直径=64 毫米，壳厚=2.6—2.4 毫米	山东莱阳王氏群上部	参看杨钟健 1954，图版 IV
Macroolithus yaotunensis	长形，较完整蛋化石一枚（V2784b) 长径=200毫米，最大直径=88 毫米，壳厚=1.8—1.4 毫米	广东南雄坪岭组	参看杨钟健 1965，图版 III
Macroolithus rugustus	长形，较完整蛋化石一枚（V2788b) 长径=180 毫米，最大直径=80毫米，壳厚=1.7—1.4 毫米	广东南雄坪岭组	参看杨钟健 1965，图版 VI
Nanhsiungoolithus chuetienensis	长形，较完整蛋化石一枚（V2782a) 长径=145毫米，最大直径=76 毫米，壳厚=1.4—1.2 毫米	广东南雄园圃组	参看杨钟健 1965，图版 X I

度看，由于这些蛋壳的厚度远小于轴的长度，也远小于转动半径。因此，可以把它看成为旋转薄壳结构，从而可以采用薄壳理论求出蛋壳中的内力和应力。

三、恐龙蛋壳的壳体微分方程式

根据壳体力学理论（Timoshenko, S. 等，1959)研究如图1表示的薄壳上一小片蛋壳 ABCD（看图2)。小块蛋壳片 ABCD 由相邻子午线和平行圆构成。该片在子午线上的位置由坐标为 θ 所确定，在平行圆上的位置由垂直于壳面的线与旋转轴间的夹角 Φ 来确定，故 Φ 和 θ 确定了壳上任何一点的位置。壳的第一曲率半径为 R_1，第二曲率半径为 R_2，平行圆半径 $R_0 = R_2\sin\Phi$。壳体平衡方程为：

$$R_1\partial N_\theta/\partial\theta + \partial(N_{\Phi\theta}R_0)/\partial\Phi + N_{\theta\Phi}R_1\cos\Phi + XR_1R_0 = 0 \qquad (1)$$

$$\partial(N_\Phi R_0)/\partial\Phi + R_1\partial N_{\theta\Phi}/\partial\theta - N_\theta R_1\cos\Phi + YR_0R_1 = 0 \qquad (2)$$

$$N_\theta/R_2 + N_\Phi/R_1 + Z = 0 \qquad (3)$$

假设恐龙蛋壳仅受有对称的压力，摩擦力很小，对蛋壳破裂不产生本质的影响，即式中的 $X = 0, N_{\theta\Phi} = N_{\Phi\theta} = 0$。上述方程可简化为：

$$N_\theta = -R_2(N_\Phi/R_1 + Z) \qquad (4)$$

$$N_\Phi = -F/(2\pi R_0\sin\Phi) \qquad (5)$$

式中：

$$F = \int_0^\Phi 2\pi R_0(Y\sin\Phi + Z\cos\Phi)R_1 d\Phi \qquad (6)$$

根据上面方程，即可解出蛋壳中的内力：$N_\Phi, N_{\Phi\theta}, N_\theta, N_{\theta\Phi}$，而应力 σ_Φ，σ_θ，便可根据 N_Φ, N_θ 求得：

$$\sigma_\Phi = N_\Phi/h,$$

$$\sigma_\theta = N_\theta/h$$

式中：h 为蛋壳厚度。

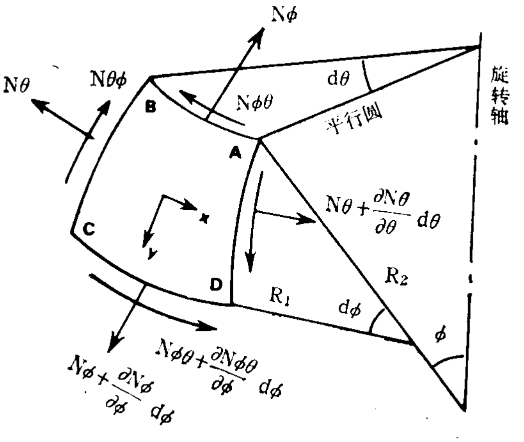


图2 小块蛋片受力图

Fig. 2 Internal forces in a small piece of eggshell

四、恐龙蛋壳中内力和应力的数值解

根据上述方程可以解出在某些特定载荷分布下的内力 N_Φ, N_θ 和应力 σ_Φ，σ_θ。假设蛋壳上受到均匀的分布载荷 p_0 的作用，如图3所示。可以用数值解法，其过程是先将微分以中心差分形式来代替：

$$dy/d\Phi = (y_{i+1} - y_{i-1})/2\Delta\Phi$$

$$d^2y/d\Phi^2 = d(dy/d\Phi)/d\Phi = (y_{i+1} - 2y_i + y_{i-1})/\Delta\Phi^2$$

由此可求出每点的三个半径：

$$R_0 = |y_i|$$

$$R_1 = |(1 - y_i'^2)|^{3/2} / |y_i''|$$

$$R_2 = R_0 / \sin \Phi_i$$

$$\mathrm{tg}\, \Phi_i = 1 / y_i'$$

$$F = \int_0^{\Phi} 2\pi R_0 (Y \sin \Phi + Z \cos \Phi) R_1 d\Phi$$

利用数值求出均匀压力下的合力F值：

$$Y = 0,\ Z = p_0,$$

$$F = \int_0^{\Phi} 2\pi p_0 R_0 R_1 \cos \Phi d\Phi$$

用C语言编写的软件对上述力学模型给出数值计算。假设在均匀外压以压强数值为10牛顿/毫米^{2}作用的情况下，沿母线方向最大内力$N_{\Phi\max}$和最大应力$\sigma_{\Phi\max}$，环向最大内力$N_{\theta\max}$和最大应力$\sigma_{\theta\max}$计算的结果列于表2。表中所有的内力单位均为牛顿/毫米（Nu/mm），应力均为牛顿/毫米^2（Nu/mm^2）。上述结果还可由图4—图7来表示。图中包括内力图和应力图二个部分，它们的纵坐标分别为内力N与应力σ的数值，横坐标表示了坐标轴Y，零点选在最大内力（或应力）的出现处。可以看出，在均匀外压作用下，N_θ，N_Φ和σ_θ，σ_Φ即内力和应力的最大值虽然随蛋壳不同外形有所变化，但这种变化也是随半径的增加而增加的。也就是说，其最大值均出现在平行圆半径最大处。因此蛋壳的这一部位最容易破坏。

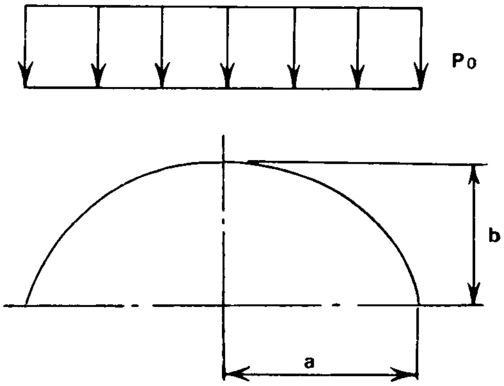


图3　蛋壳上受均匀分布压强载荷p_0作用情况

Fig. 3　Showing a case that the eggshell is subjected to uniform external lateral pressure (p_0)

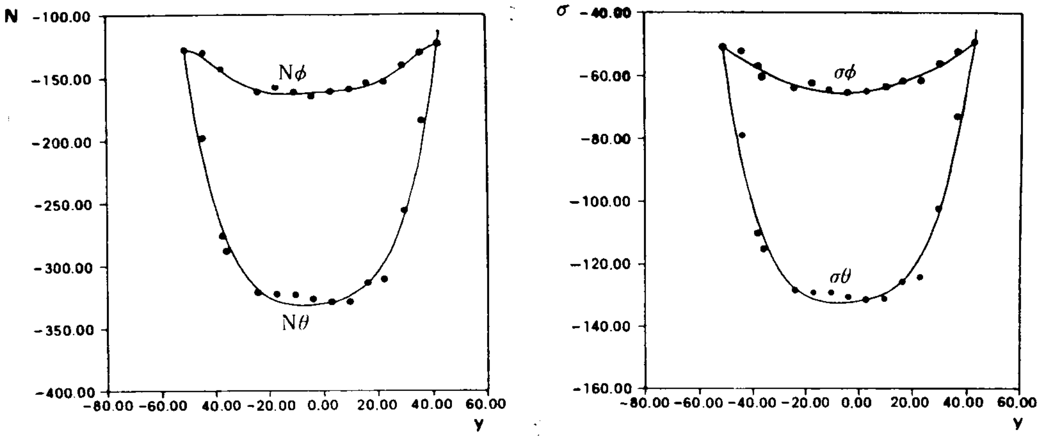
表2　四种类型恐龙蛋壳中的最大内力与最大应力数值表

Table 2　The maximal internal forces $N_{\Phi\max}$, $N_{\theta\max}$, and the maximal stresses $\sigma_{\Phi\max}$, $\sigma_{\theta\max}$ in the four groups of dinosaur eggshell

	Ovaloolithus chinkangkouensis	*Macroolithus yaotunensis*	*Macroolithus rugustus*	*Nanhsiungoolithus chuetienensis*
$N_{\Phi\max}$	163	225	215	195
$N_{\theta\max}$	330	440	395	367
壳厚h（毫米）	2.5	1.55	1.60	1.30
$\sigma_{\Phi\max}$	65	145	134	150
$\sigma_{\theta\max}$	132	284	247	282

根据上述数值计算结果，可以得到如下结论：

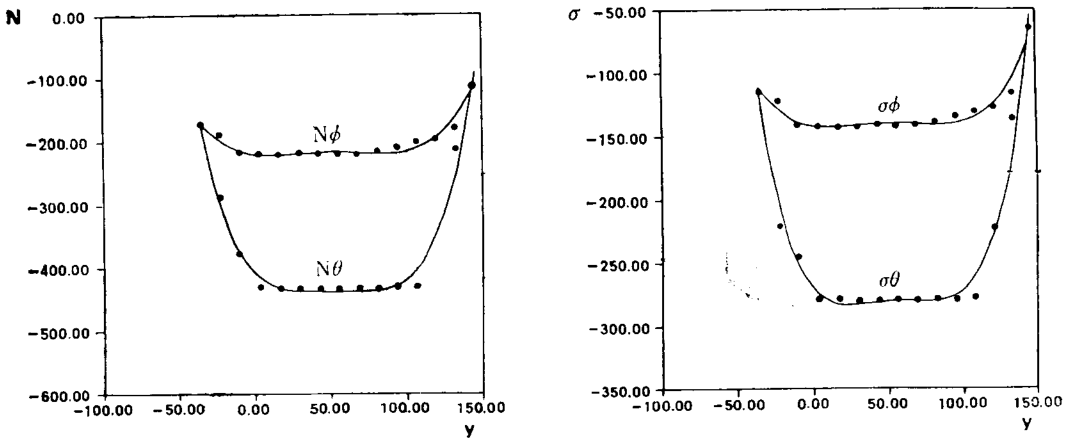
1. 在外压力的作用下，恐龙蛋壳中将出现较大的环向应力σ_θ，此应力比恐龙蛋壳沿



(a) (b)

图 4 *Ovaloolithus chinkangkouensis* 在均匀外压下的内力 (a) 和应力 (b)

Fig. 4 Internal forces (a) and stresses (b) of the eggshell in *Ovaloolithus chinkangkouensis* under the uniform external pressure



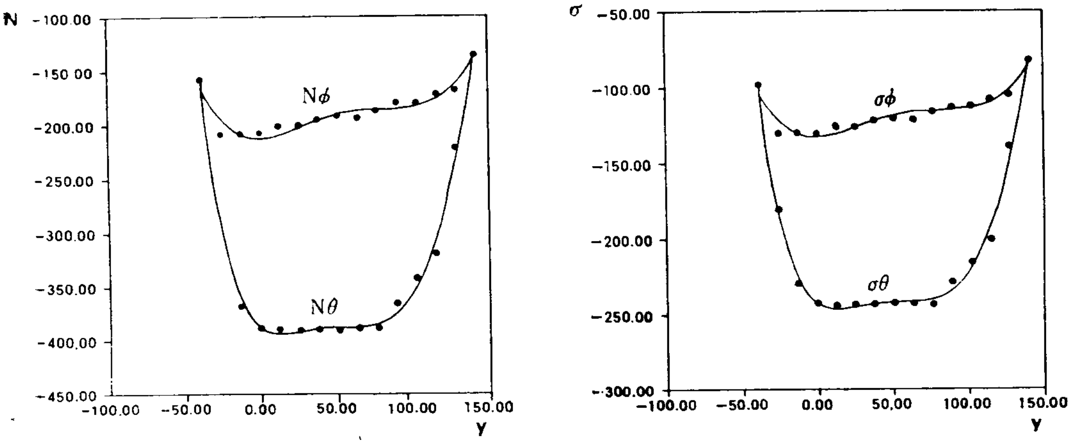
(a) (b)

图 5 *Macroolithus yaotunensis* 在均匀外压力下的内力 (a) 和应力 (b)

Fig. 5 Internal forces (a) and stresses (b) of the eggshell in *Macroolithus yaotunensis* under the uniform external pressure

母线方向的应力 σ_ϕ 约大一倍。$\sigma_{\theta\max}$ 的作用将会使蛋壳出现沿母线方向的裂纹。所以只要恐龙蛋壳沿母线和沿环向的构造型式大体相同，则蛋壳均将首先出现沿母线方向的破裂。

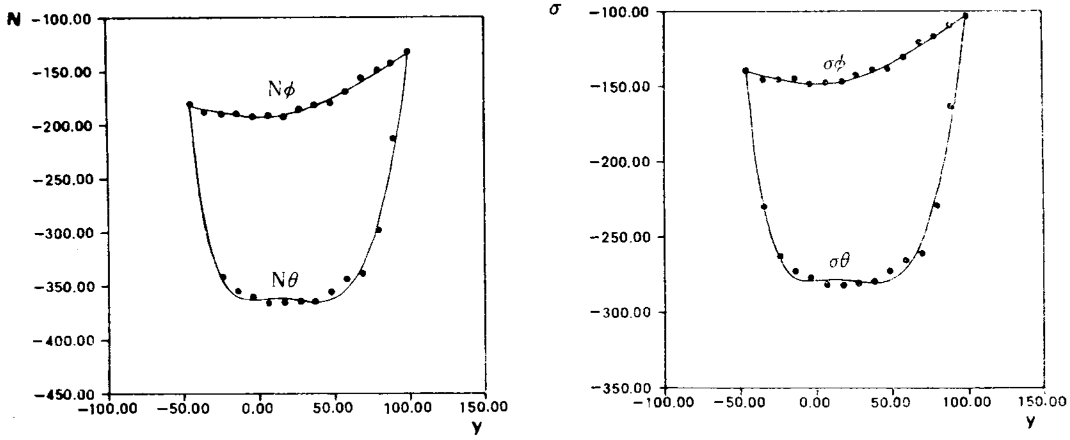
2. 在四种不同类型的恐龙蛋中，*Ovaloolithus chinkangkouensis* 的应力最小，最不易破坏；*Macroolithus yaotunensis* 和 *Nanhsiungoolithus chuetienensis* 的应力都很高，较容易破裂。



(a)　(b)

图 6　*Macroolithus rugustus* 在均匀外压力下的内力 (a) 和应力 (b)

Fig. 6　Internal forces (a) and stresses (b) of the eggshell in *Macroolithus rugustus* under the uniform external pressure



(a)　(b)

图 7　*Nanhsiungoolithus chuetuenensis* 在均匀外压力下的内力 (a) 和应力 (b)

Fig. 7　Internal forces (a) and stresses (b) of the eggshell in *Nanhsiungoolithus chuetienensis* under the uniform external pressure

3. 在同样外压压强的情况下，蛋壳最大半径与厚度之比 ($R_{\max}/h$) 值决定了 $\sigma_{\theta\max}$ 值的大小,此比值越大,则 $\sigma_{\theta\max}$ 越大,蛋壳越容易破坏。

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BIOMECHANICAL PROPERTIES OF DINOSAUR EGGSHELLS (I)

—THE STRESS ANALYSIS OF THE DINOSAUR EGGSHELLS UNDER EXTERNAL PRESSURE

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Key words Dinosaur eggshell; Thin shell; External pressure; Stress

Summary

Dinosaur are believed to be reproducing by eggs which hatch outside the body by

the heat of the sun or by the environmental temperature. Therefore, the incubation period for eggs might be longer, like that of certain living reptiles.

In recent years, several studies have indicated that the conductance of water vapor and respiratory gases, estimated from measurements of dinosaur eggshell and pore geometry, can provide evidence for the environmental conditions surrounding embryos during incubation (Seymour, 1979; Williams et al., 1984; Mou, 1992). In particular, it has been suggested from some papers about dinosaur eggshells that the last extinction of dinosaurs would be related to their reproduction (Erben, 1970; Erben et al., 1979; Zhao, 1978, 1990, 1994; Zhao, Ye et al., 1991; Zhao, Wang et al., 1993).

The primary functions of the eggshell are to protect the embryo and to mediate the interaction between the developing embryo and the outside environment. It must be strong enough to withstand the external pressure during incubation, yet fragile enough to crack easily when the young hatches. Therefore, it is clear that the mechanical properties of the eggshell have a great influence upon hatchability. There is obviously considerable potential interest in the study of biomechanical properties of the dinosaur eggshells.

A list of the material available for the present study is given in Table 1. Geometrically, it is to consider the eggshell as the shell of revolution shaped by the curve ABC, called generatrix, moving around the axis AB (Fig. 1). In this case, internal forces and stress in the eggshell can be calculated by the theory of thin shell (Fig. 2). The equilibrium conditions of the shell are represented by the following equations:

$$R_1\partial N_\theta/\partial\theta + \partial(N_{\Phi\theta}R_0)/\partial\Phi + N_{\theta\Phi}R_1\cos\Phi + XR_1R_0 = 0 \tag{1}$$

$$\partial(N_\Phi R_0)/\partial\Phi + R_1\partial N_{\theta\Phi}/\partial\theta - N_\theta R_1\cos\Phi + YR_0R_1 = 0 \tag{2}$$

$$N_\theta/R_2 + N_\Phi/R_1 + Z = 0 \tag{3}$$

where, R_1 is the first radius of curvature; R_2 is the second radius of curvature; R_0 is $R_2\sin\Phi$.

Assuming that the dinosaur eggshell is only subjected to the symmetrical pressure, the frictional force can be neglected, i. e., put $X = 0$, $N_{\theta\varphi} = N_{\varphi\theta} = 0$。 Thus above equation becomes

$$N_\theta = -R_2(N_\varphi/R_1 + Z) \tag{4}$$

$$N_\Phi = -F/(2\pi R_0\sin\Phi) \tag{5}$$

where
$$F = \int_0^\Phi 2\pi R_0(Y\sin\Phi + Z\cos\Phi)R_1 d\Phi \tag{6}$$

Hence, internal forces N_φ, N_θ and $N_{\varphi\theta} = N_{\theta\varphi}$ can be obtained from Eqs (1)—(3), or according to Eqs (4)—(6) under the case of the symmetrical pressure. Stress σ_φ, σ_θ can be obtained from N_φ, N_θ:

$$\sigma_\varphi = N_\varphi/h, \quad \sigma_\theta = N_\theta/h,$$

in which h is thickness of the eggshell.

In the case of distributive load p_0 (Fig. 3), solution can be obtained by numerical method.

First, differential is instead of centre difference:

$$dy/d\Phi = (y_{i+1} - y_{i-1})/2\Delta\Phi$$

$$d^2y/d\Phi^2 = d(dy/d\Phi)/d\Phi = (y_{i+1} - 2y_i + y_{i-1})/\Delta\Phi^2$$

Then three radii R_0, R_1 and R_2 at every point can be deduced:

$$R_0 = |y_i|$$

$$R_1 = |(1 - y_i'^2)|^{3/2}/|y_i''|$$

$$R_2 = R_0/\sin\Phi_i$$

in which Φ_i is expressed from

$$\mathrm{tg}\,\Phi_i = 1/y_i'$$

and integral

$$F = \int_0^{\Phi} 2\pi R_0(Y\sin\Phi + Z\cos\Phi)R_1 d\Phi$$

In the case of uniform external lateral pressure

$$Y = 0,\ Z = p_0,\ F = \int_0^{\Phi} 2\pi p_0 R_0 R_1 \cos\Phi d\Phi$$

With software composed in Languae C the numerical solution for the mechanics model can be obtained. Assuming the case that the eggshell is subjected to external pressure 10 Nu/mm^2, maximal internal forces $N_{\Phi\max}$, $N_{\theta\max}$, and maximal stresses $\sigma_{\Phi\max}$, $\sigma_{\theta\max}$ are written in Table 2. Curves of N_Φ, N_θ, σ_Φ and σ_θ are also expressed in figures 4—7. Here, the internal forces N_Φ, N_θ and the stresses σ_Φ, σ_θ are expressed by ordinate, and the position of points, by the abscissa. Origins of coordinates are chosen in the points where internal forces of stresses obtain maximal value.

Based upon the foregoing calculation, the following conclusions may be drawn:

1. Under the external pressure, the dinosaur eggshell produces much greater circular stress σ_θ which is as much again as the stress σ_Φ producing along the generatrix direction. Consequently, the crack in the eggshell, caused by such action of $\sigma_{\theta\max}$, takes place in the generatrix direction.

2. In the four groups of dinosaur eggshell, the stress of ***Ovaloolithus chinkangkouensis*** is the smallest, and the eggshell is hard to crack. On the contrary, the stress in ***Macroolithus yaotunensis*** and ***Nanhsiungoolithus chuetienensis*** is greater than the former, and the eggshell is liable to crack.

3. $\sigma_{\theta\max}$ is determined by the ratio of $R_{\max}/h$. The greater this ratio, the greater is $\sigma_{\theta\max}$, and the easier the eggshell to crack.