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# 恐龙蛋壳的生物力学性质（II）——在外力作用下恐龙蛋壳的两种可能破坏形式[1)]

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**关键词** 恐龙蛋 薄壳 外压 应力

**内容提要**

在外力作用下，以 *Macroolithus yaotunensis*, *Macroolithus rugustus* 和 *Nanhsiungoolithus chuetienensis* 为代表的三种蛋壳可能出现外压失稳而破坏；以 *Ovaloolithus chinkangkouensis* 为代表的，由于临界外压值较大，不能肯定是失稳的形式发生破坏，也有可能存在以受压破损形式发生破坏的可能性。这一成果为进一步分析不同类型恐龙蛋在蛋窝中不同的排列方式，揭示由这些蛋化石为代表的恐龙生殖行为的奥秘提供可靠依据。

## 一、引言

根据生物力学的基本理论和分析方法研究白垩纪恐龙蛋壳结构的力学性质是一项新的尝试。赵资奎等(1994)提出，不同种的恐龙蛋，虽然其形状各色各样，但大体上为旋转对称外形。这些蛋的蛋壳厚度远小于轴的长度，也远小于转动半径，因此，可把它们看成为旋转薄壳，可以采用薄壳理论求出这些恐龙蛋壳中的内力和应力。

本文是上一篇报告(赵资奎等，1994)的继续，通过对晚白垩世四种不同类型恐龙蛋壳的受力特性和变形特性的力学分析，可以进一步加深对蛋壳结构的演化、不同类群恐龙蛋在蛋窝中不同的排列方式以及恐龙最后绝灭问题的理解；也有可能为生物力学和古脊椎动物学开辟一个新的研究领域。

本文研究的蛋化石标本见赵资奎等(1994)表1，为了便于用统一的数学方法进行研究，将这四种恐龙蛋化石标本编号如表1。

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**表 1 本文研究的蛋化石标本**

**Table 1 The dinosaur eggs analyzed in this study**

| 蛋化石名称 / 蛋型组别 / 外壳形状 | *Ovaloolithus chinkangkouensis* | *Macroolithus yaotunensis* | *Macroolithus rugustus* | *Nanhsiungoolithus chuetienensis* |
|---|---|---|---|---|
| | A | B | C | D |
| 标准椭球型壳 Standard contour of eggshell | A01<br>A02<br>A03 | B01<br>B02<br>B03 | C01<br>C02<br>C03 | D01<br>D02<br>D03 |
| 实际外形壳 Real contour of eggshell | A11<br>A12<br>A13 | B11<br>B12<br>B13 | C11<br>C12<br>C13 | D11<br>D12<br>D13 |

## 二、恐龙蛋壳的平衡方程

从外观上看,恐龙蛋壳的形状与鸟蛋壳的一样为旋转薄壳,也就是说相当于一个曲线(在此称为母线)绕一轴线(称为旋转轴或对称轴)旋转而形成。母线至旋转轴的距离称为平行圆半径(图 1),以 $R_0$ 表示。

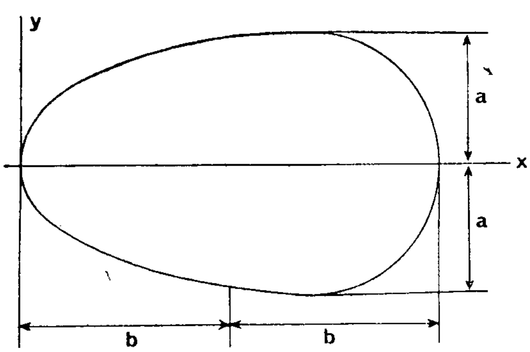


图 1 恐龙蛋的旋转模型

Fig. 1 Rotational thin shell model of dinosaur egg.

恐龙蛋壳是两端稍厚而中间较薄,这是蛋壳在输卵管中形成时由于输卵管的运动而造成端部收缩的结果。因为蛋壳的中部是受力危险部位,并且其厚度变化不大,所以在这一部位取蛋壳的平均厚度来进行研究。由于蛋壳的厚度远小于壳的最小曲率半径,因此可以当作薄壳来研究,得到其平衡微分方程式(Timoshenko 等,1959):

$$
\begin{aligned}
&R_1 \frac{\partial N_\theta}{\partial \theta} + \frac{\partial (N_{\Phi\theta} R_0)}{\partial \Phi} + N_{\theta\Phi} R_1 \cos\Phi + X R_1 R_2 = 0, \\
&\frac{\partial (N_\Phi R_0)}{\partial \Phi} + R_1 \frac{\partial N_{\theta\Phi}}{\partial \theta} - N_\theta R_1 \cos\Phi + Y R_0 R_1 = 0, \qquad (1) \\
&\frac{N_\theta}{R_2} + \frac{N_\Phi}{R_1} + Z = 0。
\end{aligned}
$$

# 三、应 力 计 算

恐龙是把卵产在质地松软的沙土中的，可以认为这些蛋所受到的外力是均匀分布压力，即 $N_{\varphi\theta} = N_{\theta\varphi} = 0$，对于许多恐龙蛋形状的数据，经过处理便可得到无因次外形曲线，以 $A$ 型为例为：

$$\bar{y} = 2.66\bar{x}^5 - 0.868\bar{x}^4 - 6.25\bar{x}^3 + 4.45\bar{x}^2, \tag{2}$$

式中

$$\bar{y} = y/b, \tag{3a}$$

$$\bar{x} = x/a。 \tag{3b}$$

用此表示方法即可排除蛋壳大小的尺寸因素影响而便于用统一的数学方法进行研究。

采用数值方法将微分以中心差分的形式替代：

$$\frac{dy}{d\Phi} = \frac{y_{i+1} - y_{i-1}}{2\Delta\Phi}, \tag{4a}$$

$$\frac{d^2y}{d\Phi^2} = \frac{d}{d\Phi}\left(\frac{dy}{d\Phi}\right) = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta\Phi^2}。 \tag{4b}$$

由此得到壳体三个曲率半径的差分表达式：

$$R_1 = \left|1 - \left(\frac{y_{i+1} - y_{i-1}}{2\Delta\Phi}\right)^2\right|^{\frac{3}{2}} \Big/ \left|\frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta\Phi^2}\right|, \tag{5a}$$

$$R_2 = R_0/\sin\Phi, \tag{5b}$$

$$R_0 = |y_i| \tag{5c}$$

以及

$$\mathrm{tg}\,\Phi_i = \frac{2\Delta\Phi}{y_{i+1} - y_{i-1}}。 \tag{6}$$

采用无因次参数

$$\bar{N}_\Phi = \frac{N_\Phi}{pb} \tag{7a}$$

及

$$\bar{N}_\theta = \frac{N_\theta}{pb}, \tag{7b}$$

式中 $\bar{N}_\Phi, \bar{N}_\theta$ 分别为壳内沿母线方向和圆周方向的无因次内力。$N_\Phi$、$N_\theta$ 分别为上述两个方向的内力（即壳体单位长度上的值），$p$ 是分布压力。由计算可得到不同类型恐龙蛋壳的 $\bar{N}_\Phi, \bar{N}_\theta$ 值的曲线（赵资奎等，1994），根据所求点的厚度得到该点无因次应力 $\bar{\sigma}_\Phi, \bar{\sigma}_\theta$ 及应力 $\sigma_\Phi$、$\sigma_\theta$：

$$\bar{\sigma}_\Phi = \frac{\sigma_\Phi h}{pb} = \frac{\bar{N}_\Phi}{h}, \tag{8a}$$

$$\bar{\sigma}_\theta = \frac{\sigma_\theta h}{pb} = \frac{\bar{N}_\theta}{h}, \tag{8b}$$

式中 $\Phi$ 为母线方向，$\theta$ 为垂直于母线方向。$h$ 为研究点壳的厚度。

可以看出壳中应力的大小取决于 $pb/h$ 值。此值越大则应力越大。由图2还可见到，$N_\Phi$ 及 $N_\theta$ 及由它们决定的 $\sigma_\Phi$、$\sigma_\theta$ 最大值均出现于蛋壳的中间部分，并有 $\sigma_\theta > \sigma_\Phi$，$N_\theta > N_\Phi$。也就是说首先会在壳的中部出现沿母线方向的裂缝。从已发现的那些原来完整的恐龙蛋但在石化过程中蛋壳受压破裂的裂纹可以看出(参看杨钟健，1965，图版 III，V，VI，VIII，XI 和 XII)，蛋壳破裂纹的形式与本文的研究结果相一致。

## 四、蛋壳的受压失稳分析

恐龙蛋作为一个薄壳结构在分布的外界压力作用下有可能出现外压失稳情况。其失稳图形如图2所示。在失稳的基础上将产生向内的凹陷破坏，破坏线沿母线方向。这与已发现的蛋化石破裂形式(如图3)相同。

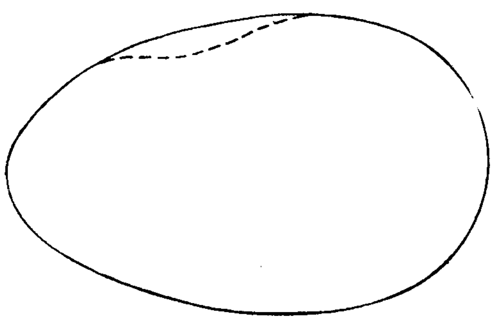

图2 恐龙蛋外压失稳示意图

**Fig. 2** Diagram showing instability(buckling) of dinosaur egg under external pressure

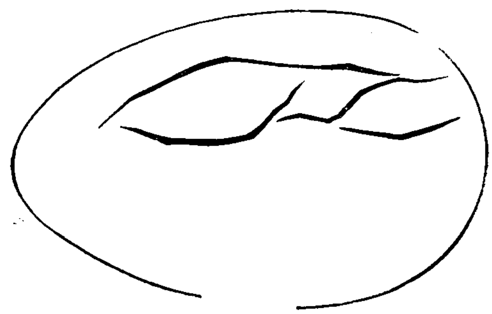

图3 恐龙蛋破裂形式图

**Fig. 3** Diagram of breaking type of dinosaur egg

失稳的临界外压强无因次量 $\bar{p}_{cr}$ 的公式可以得:

$$\bar{p}_{cr} = \frac{P_{cr}ab^2}{Eh^3} = \frac{K_y\pi^2}{12(1-\mu^2)}, \tag{9}$$

式中 $E$ 为材料的压缩弹性模量，$\mu$ 为材料某个方向变形与同它垂直方向变形之间的比例，称为泊松比。$K_y$ 为一系数，它取决于壳的几何形状，可以由图4查得 (Column Research Committee of Japan, 1971)。

图4中的横坐标 $Z_L = b^2(\sqrt{1-\mu^2}/\sqrt{ab})$。

此式中的 $a$ 和 $b$ 分别为壳的半短轴与半长轴。由图4根据求得的 $Z_L$ 查出 $K_y$ 以后即可由(9)式求出实际临界压强 $p_{cr}$。

以 $A$、$B$、$C$、$D$ 四种实际形状的恐龙蛋壳为例来计算其临界压强 $p_{cr}$。恐龙蛋壳主要是由微晶方解石和少量胶原纤维组成。从固体力学角度可以把它看成是一种复合材料结

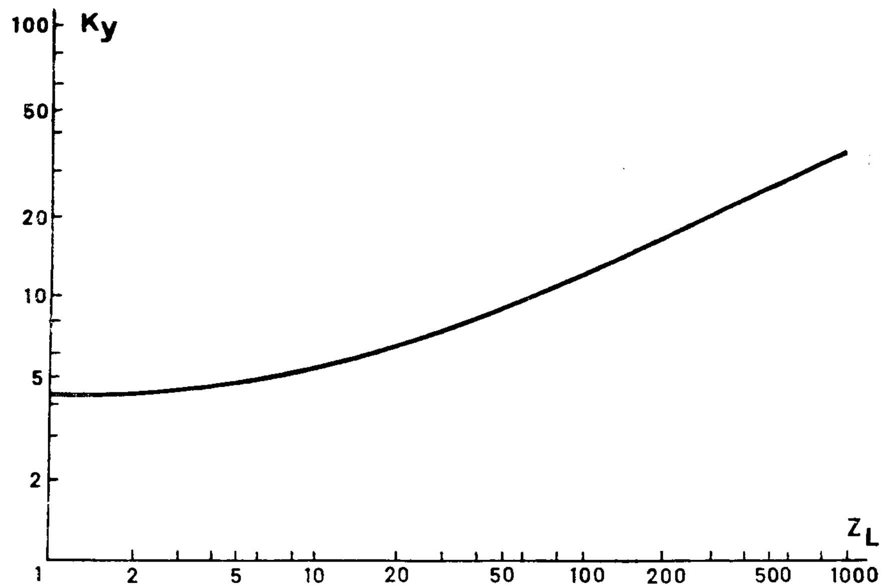


图 4 系数 $K_Y$ 曲线

Fig. 4 Curve of *Ky* coefficient

**表 2 四种形式恐龙蛋壳临界压强值**

**Table 2 Critical pressure in the four types of dinosaur eggshell**

| 蛋 型 | A | B | C | D |
|---|---|---|---|---|
| b(cm) | 4.7 | 10.0 | 9.0 | 7.3 |
| a(cm) | 3.2 | 4.4 | 4.0 | 3.8 |
| h(cm) | 0.24 | 0.14 | 0.14 | 0.12 |
| $Z_L$(cm) | 24.4 | 123.4 | 104.8 | 76.4 |
| $K_Y$(cm) | 7.0 | 16.0 | 15.0 | 11.0 |
| $p_{cr}$(MPa) | 6.01 | 0.44 | 0.56 | 0.41 |
| $p_{cr}$(kg/cm$^2$) | 58.8 | 4.29 | 5.48 | 4.03 |
| 抗失稳能力 resistance to unstability | 最强 | 第三 | 其次 | 最差 |

构，取弹性模量 $E = 5.0\ GP_a$，泊松比 $\mu = 0.25$，利用前述公式可以求出 $p_{cr}$ 值，见表 2。

## 五、结 论

如果将上述使蛋壳失稳的压力 $p_{cr}$ 作为分布外力 $p$ 加到壳上，则壳内应力为 $\sigma_\theta$ 可以由式 8 得到（表 3）。

由上述情况来看，以 *Macroolithus yaotunensis*, *Macroolithus rugustus* 和 *Nanhsi-*

表 3 四种型式恐龙蛋壳内的应力
Table 3 Stresses in the four types of dinosaur eggshell

| 蛋 型 | A | B | C | D |
|---|---|---|---|---|
| 应力 (MPa) Stress | 56.6 | 12.6 | 12.6 | 10.0 |
| 应力 (kg/cm²) Stress | 555.0 | 123.0 | 123.0 | 98.3 |

*ungoolithus chuetienensis* 为代表的,即 B、*C*、*D* 型三种恐龙蛋,在很低的应力下,可能会发生蛋壳在外压下失稳而破坏,但对于以 *Ovaloolithus chinkangkouensis* 为代表的 *A* 型恐龙蛋来说,则由于临界外压较大,不能肯定是失稳的形式发生破坏,即也存在以受压破损形式发生破坏的可能性。

本文插图由沈文龙先生绘制,在此表示感谢。

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# BIOMECHANICAL PROPERTIES OF DINOSAUR EGGSHELLS (II)——TWO BREAKING TYPES OF THE DINOSAUR EGGSHELLS UNDER EXTERNAL PRESSURE

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**Key words** Dinosaur eggshell; Thin shell; External pressure; Stress

## Summary

An attempt has been made to analyse mechanical properties of dinosaur eggshells

with the basic theory and methods of biomechanics. Zhao et al. (1994) advanced that the dinosaur eggshell can be considered as the rotational thin shell, that is to say, it is formed by rotating a curve (generant line) moving around an axis (rotational axis or symmetric axis). The distance between any point of the generant line and the rotational axis is named parallel circle radius of this point, expressed by $R_0$. According to thin shell theory (Timoshenko et al., 1959), the equilibrium equations of the eggshell can be written as

$$R_1 \frac{\partial N_\theta}{\partial \theta} + \frac{\partial (N_{\Phi\theta} R_0)}{\partial \Phi} + N_{\theta\Phi} R_1 \cos\Phi + X R_1 R_2 = 0,$$

$$\frac{\partial (N_\Phi R_0)}{\partial \Phi} + R_1 \frac{\partial N_{\theta\Phi}}{\partial \theta} - N_\theta R_1 \cos\Phi + Y R_0 R_1 = 0, \tag{1}$$

$$\frac{N_\theta}{R_2} + \frac{N_\Phi}{R_1} + Z = 0.$$

The dinosaurs bury their eggs in sand or earth for incubation. Assuming that exerting external forces on these eggs within the nest are equal distribution pressure, i. e. $N_{\Phi\theta} = N_{\theta\Phi} = 0$. After treating the statistical figures of the dinosaur eggshells, a non-dimensional contour curve formula can be obtained:

$$\bar{y} = 2.66\bar{x}^5 - 0.868\bar{x}^4 - 6.25\bar{x}^3 + 4.45\bar{x}^2, \tag{2}$$

in which

$$\bar{y} = y/b, \tag{3a}$$

$$\bar{x} = x/a. \tag{3b}$$

Using numerical method substitutes the differential with central difference:

$$\frac{dy}{d\Phi} = \frac{y_{i+1} - y_{i-1}}{2\Delta\Phi}, \tag{4a}$$

$$\frac{d^2y}{d\Phi^2} = \frac{d}{d\Phi}\left(\frac{dy}{d\Phi}\right) = \frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta\Phi^2}. \tag{4b}$$

Thus the three difference expressions of curvature radius of the eggshell can be obtained:

$$R_1 = \left|1 - \left(\frac{y_{i+1} - y_{i-1}}{2\Delta\Phi}\right)^2\right|^{\frac{3}{2}} \Big/ \left|\frac{y_{i+1} - 2y_i + y_{i-1}}{\Delta\Phi^2}\right|, \tag{5a}$$

$$R_2 = R_0 / \sin\Phi, \tag{5b}$$

$$R_0 = |y_i| \tag{5c}$$

and

$$\mathrm{tg}\,\Phi_i = \frac{2\Delta\Phi}{y_{i+1} - y_{i-1}} \tag{6}$$

Using nondimension parameters:

$$\bar{N}_\Phi = \frac{N_\Phi}{pb}, \tag{7a}$$

$$\overline{N}_\theta = \frac{N_\theta}{pb}, \quad (7b)$$

in which $\overline{N}_\Phi$ and $\overline{N}_\theta$ are nondimensional internal forces along the generant line direction and the circumference direction, respectively; $N_\Phi$ and $N_\theta$ are internal forces per unit length along the two directions; p expresses pressure. The curves of $N_\Phi$ and $N_\theta$ in the different types of dinosaur eggshell can be obtained (Zhao et al., 1994). Thus, the nondimension stresses $\bar{\sigma}_\Phi$, $\bar{\sigma}_\theta$ and stresses $\sigma_\Phi$, $\sigma_\theta$ in each point on the eggshell can be calculated by the following formulae:

$$\bar{\sigma}_\Phi = \frac{\sigma_\Phi h}{pb} = \frac{\overline{N}_\Phi}{h}, \quad (8a)$$

$$\bar{\sigma}_\theta = \frac{\sigma_\theta h}{pb} = \frac{\overline{N}_\theta}{h}, \quad (8b)$$

where, $\varphi$ and $\theta$ express the direction of generant line and the circumference direction respectively; h is thickness of the eggshell.

From this, it can be seen that the stresses in the eggshell are in proportion to the value of pb/h, and that both of $N_{\Phi\max}$,$N_{\theta\max}$ and $\sigma_{\Phi\max}$,$\sigma_{\theta\max}$ arise in the equatorial region of the egg (Fig. 2), and $\sigma_\theta > \sigma_\Phi$, $N_\theta > N_\Phi$. That is to say, cracks of the eggshell caused by external pressure should first emerge in the equatorial region along the direction of generant line. It subsides and breaks under its instability (buckling) condition, as shown in figures 2 and 3. From the crack patterns on the better preserved dinosaur eggshells (see Plate III, V, VI, VIII,XI and XII of Young, 1965), it seems that the breaklines of the eggshells triggered by the external pressure during their fossilization initially produced along the direction of generant line, which are consistent with the present study.

The buckling formula of nondimensional value $\bar{P}_{cr}$ under the critical external pressure can be obtained:

$$\bar{p}_{cr} = \frac{p_{cr}ab^2}{Eh^3} = \frac{K_y\pi^2}{12(1-\mu^2)}, \quad (9)$$

where, E is compressive elastic modulus; $\mu$ is the ratio of material deformation in one direction to material deformation in its vertical direction, called Poisson's ratio; $K_y$ is a coefficient which depends on geometric shape of the eggshell. It can be read up from Figure 4 (Column Research Committee of Japan, 1971).

The abscissa $Z_L = b^2\sqrt{1-\mu^2}/\sqrt{ab}$, in which a and b are the lengths of half a short axis and half a long one, respectively. After calculating $Z_L$, $K_y$ can be found out from Figure 4. Finally, the critical pressure $P_{cr}$ can be obtained from the expression (9).

The dinosaur eggshell is not a homogeneous structure. It is compounded of two quite different materials: calcitic crystallites and a small amount of organic matrix composed of a collagen-like protein. Therefore, it can be regarded as the structure of composite material. Taking its elastic modulus $E = 5.0$GPa, Poisson's ratio $\mu = 0.25$, using the above expressions and curves, $P_{cr}$ can be obtained (Table 2).

If we take the pressure $P_{cr}$ of the eggshell instability as the external distribution forces and exert them on the eggshell, the stress $\sigma_\theta$ in it can be calculated from the expression (8), as shown in Table 3.

From the foregoing results, we can see that in the low stress level, the eggshell, represented by *Macroolithus yaotunensis*, *Macroolithus rugustus* and *Nanhsiungoolithus chuetienensis*, could be broken due to the instability under the external pressure. But that of *Ovaloolithus chinkangkouensis* could have the possibility of compressive damage.

# 更　　正

本刊第 32 卷第 3 期封二刊登的第六届中生代陆相生态系统会议日期应为 1995 年 9 月 1 日至 9 月 4 日，特此更正。

本刊编辑部